KIRAN TUTORIALS

Seat No.

Std 10 (English) **Mathematics Part - II** **Date 01-10-20**

Time 1HRS **Chapter 6.,6.00** **Marks 20**

Q.1 Multiple Choice Questions **1**

1 $\frac{\tan^2\theta}{1 + \tan^2\theta}$ is equal to

a. 2 $\sin^2\theta$ b. 2 $\cos^2\theta$ c. $\sin^2\theta$ d. $\cos^2\theta$

Ans Option c.

Q.2 Answer the following. **1**

1 Prove the following

$\cos^2\theta\,(1 + \tan^2\theta) = 1$

Ans LHS $=\cos^2\theta\,(1 + \tan^2\theta)$

$=\cos^2\theta \times \sec^2\theta$ ...$[1 + \tan^2\theta = \sec^2\theta]$

$= \cos^2\theta \times \frac{1}{\cos^2\theta}$ ...$[\sec\theta = \frac{1}{\cos\theta}]$

$= 1$

$\therefore$ LHS = RHS

$\therefore$ $\cos^2\theta\,(1 + \tan^2\theta) = 1$

Q.3 Answer the following (Any One) **2**

1 Prove the following

$\frac{\tan^3\theta - 1}{\tan\theta - 1} = \sec^2\theta + \tan\theta$

Ans LHS $= \frac{\tan^3\theta - 1}{\tan\theta - 1}$ [Using $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

$= \frac{(\tan\theta)^3 - 1^3}{\tan\theta - 1}$

$= \frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta \times 1 + 1^2)}{\tan\theta - 1}$

$= \tan^2\theta + \tan\theta + 1$

$= \sec^2\theta + \tan\theta$ ... $[1 + \tan^2\theta = \sec^2\theta]$

$\therefore$ LHS = RHS

$\therefore$ $\frac{\tan^3\theta - 1}{\tan\theta - 1} = \sec^2\theta + \tan\theta$

2 Prove that $\sec\theta + \tan\theta = \frac{\cos\theta}{1 - \sin\theta}$

Ans $\sec\theta + \tan\theta = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

$= \frac{1 + \sin\theta}{\cos\theta}$

$= \frac{(1 + \sin\theta)(1 - \sin\theta)}{\cos\theta\,(1 - \sin\theta)}$

$= \frac{1^2 - \sin^2\theta}{\cos\theta\,(1 - \sin\theta)}$

$= \frac{\cos^2\theta}{\cos\theta\,(1 - \sin\theta)}$

$\sec\theta + \tan\theta$

$= \frac{\cos\theta}{1-\sin\theta}$

Q.4 **Solve the following** **6**

1 Prove the following

$\cot^2\theta - \tan^2\theta = \mathrm{cosec}^2\theta - \sec^2\theta$

Ans LHS = $\cot^2\theta - \tan^2\theta$

$$= \left(\frac{\cos\theta}{\sin\theta}\right)^2 - \left(\frac{\sin\theta}{\cos\theta}\right)^2 \quad \ldots \left[\cot\theta = \frac{\cos\theta}{\sin\theta},\ \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$= \frac{\cos^2\theta}{\sin^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{\cos^4\theta - \sin^4\theta}{\sin^2\theta \times \cos^2\theta}$$

$$= \frac{(\cos^2\theta)^2 - (\sin^2\theta)^2}{\sin^2\theta \times \cos^2\theta}$$

$$= \frac{(\cos^2\theta + \sin^2\theta) \times (\cos^2\theta - \sin^2\theta)}{\sin^2\theta \times \cos^2\theta} \quad \ldots [(a+b)(a-b) = a^2 - b^2]$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta \times \cos^2\theta}$$

$$= \frac{\cos^2\theta}{\sin^2\theta \times \cos^2\theta} - \frac{\sin^2\theta}{\sin^2\theta \times \cos^2\theta}$$

$$= \frac{1}{\sin^2\theta} - \frac{1}{\cos^2\theta}$$

$$= \mathrm{cosec}^2\theta - \sec^2\theta \quad \ldots \left[\mathrm{cosec}\,\theta = \frac{1}{\sin\theta},\ \sec\theta = \frac{1}{\cos\theta}\right]$$

$\therefore$ LHS = RHS

2 Prove that $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

Ans **Proof:** LHS = $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$$= \frac{\sin\theta\,(1 - 2\sin^2\theta)}{\cos\theta\,(2\cos^2 - 1)}$$

$$= \frac{\sin\theta\,(\sin^2\theta + \cos^2\theta - 2\sin^2\theta)}{\cos\theta\,(2\cos^2\theta - \sin^2\theta - \cos^2\theta)} \quad \ldots (\sin^2\theta + \cos^2\theta = 1)$$

$$= \frac{\sin\theta\,(\cos^2\theta - \sin^2\theta)}{\cos\theta\,(\cos^2\theta - \sin^2\theta)}$$

$$= \frac{\sin\theta}{\cos\theta}$$

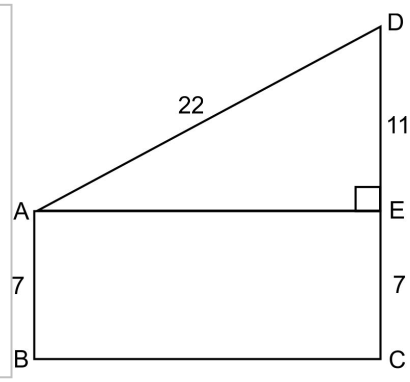
$= \tan\theta$ = RHS

$\therefore$ $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

Q.5 **Answer the following(Any One)** **4**

1 Two poles of height 18 meters and 7 meters are erected on the ground. A wire of length 22 metres ties the two tops of poles. Find the angle made by the wire the horizontal.

Ans



Seg AB and seg CD represent 2 pole erected on the ground BC.

AB = Height of one pole = 7 m.

CD = Height of 2nd pole = 18 m.

Seg AD represents the wire which ties the tops of 2 poles.

∴ AD = length of the wire = 22 m.

Seg AE represents the horizontal.

∠ DAE = Angle made wire, the wire with the horizontal.

□ ABCD is a rectangle.

∴ AB = CE = 7 m. [Opposite sides of a rectangle are equal]

CD = CE + DE (C – E – D)

∴ 18 = 7 + DE

∴ DE = 18 – 7

∴ DE = 11 m.

In right angled Δ AED,

∴ sin ∠DAE = $\frac{DE}{AD}$

∴ sin ∠DAE = $\frac{11}{22}$

∴ sin ∠DAE = $\frac{1}{2}$

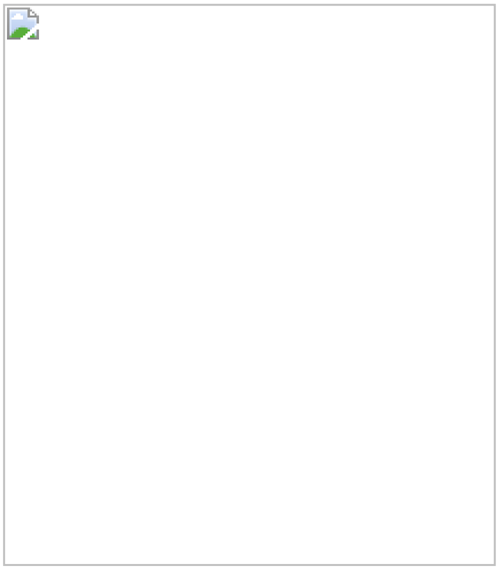
But, sin 30º = $\frac{1}{2}$ (S. T. R.)

∴ ∠DAE = 30º

Ans. : The angle made by the wire with the horizontal is 30º.

2 Two buildings are facing each other on a road of width 12 metre. From the top of the first building, which is 10 metre high, the angle of elevation of the top of the second is found to be 60°. What is the height of the second building ?

Ans



Let AB and DC are two buildings.

∴ AB = 10 m.

AD is the width of road. ∴ AD = 12 m.

Draw Seg BE $\perp$ Seg DC such that C-E-D.

∴	$m\angle EBC = 60^\circ$	[angle of elevation]
□	ABED is a rectangle.	[Each angle is 90°]
∴	AB = DE = 10 m and AD = BE = 12 m	[opposite sides of rectangle are congruent]

In right angled Δ CEB,

$\tan 60^\circ = \frac{CE}{BE}$

∴ $\sqrt{3} = \frac{CE}{12}$

CD = CE + ED [C - E - D]

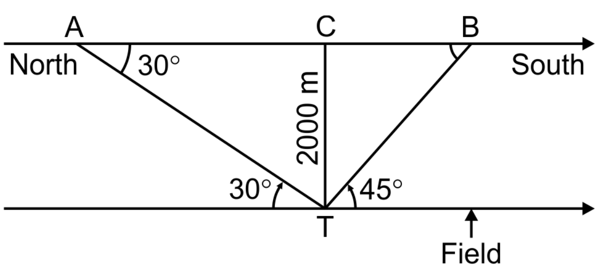
$= 12\sqrt{3} + 10 = 12 \times 1.73 + 10$

∴ CD = 30.76

∴ Height of the second building is 30.76 m

Q.6 **Creative questions** **3**

1 Construct a trigonometry word problem (find speed) by looking at the figure . Solve the problem you have constructed.



Ans **Question**: A bird was flying in a line parallel to the ground from north to south at a height of 2000 metres. Tom, standing in the middle of the field, first observed the bird in the north at an angle of 30°. After 3 minutes, he again observed it in the south at an angle of 45°. Find the speed of the bird in km/h.

solution: Let T be the position of Tom. A is the initial position of the bird.

B is the final position of the bird. CT = 2000 m.

$$\left.\begin{aligned}\angle CAT &= 30^\circ\\ \angle CBT &= 45^\circ\end{aligned}\right\}$$ ... (Alternate angles)

In $\triangle$ CAT, tan∠CAT = tan 30° = $\frac{CT}{AC}$

$\therefore \quad \frac{1}{\sqrt{3}} = \frac{2000}{AC}$

$\therefore \quad$ AC = $2000\sqrt{3}$ = 2000 × 1.73 = 3460 m ... (1)

In $\triangle$ CBT, tan ∠CBT = tan 45° = $\frac{CT}{BC}$

$\therefore \quad 1 = \frac{2000}{BC}$

$\therefore \quad$ BC = 2000 m ... (2)

From (1) and (2),

AB = AC + CB = (3460 + 2000) m = 5460 m

So, the distance covered by the bird in 3 minutes = 5460 m = $\frac{5460}{1000}$km

$\therefore \quad$ the distance covered by bird in 1 hour (60 minutes)

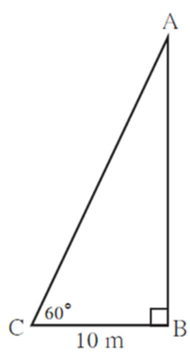
$= \frac{5460}{1000 \times 3} \times 60 = \frac{5460 \times 20}{1000} \frac{km}{h}$ = 109.2 km/h

The speed of the bird is **109.2** km/h

Q.7 Answer the following (Any One) 3

1 An observer at a distance of 10 m from a tree looks at the top of the tree, the angle of elevation is 60°. What is the height of the tree? ($\sqrt{3}$ = 1.73)

Ans



In figure, AB = h = height of the tree.

BC = 10 m, distance of the observer from the tree .

Angle of elevation (θ) = ∠BCA = 60°

from figure, tanθ = $\frac{AB}{BC}$ ... (I)

tan 60° = $\sqrt{3}$ ... (II)

$\therefore \quad \frac{AB}{BC} = \sqrt{3}$ ... from equation (I)and (II)

$\therefore \quad$ AB = BC $\sqrt{3}$ = 10 $\sqrt{3}$

$\therefore \quad$ AB = 10 × 1.73 = 17.3 m

$\therefore \quad$ height of the tree is 17.3m.

2 Prove the following.

$$\frac{1}{\sin A + \cos A + 1} + \frac{1}{\sin A + \cos A - 1} = \sec A + \operatorname{cosec} A$$

Ans L.H.S.= $\frac{1}{\sin A + \cos A + 1} + \frac{1}{\sin A + \cos A - 1}$

$= \frac{\sin A + \cos A - 1 + \sin A + \cos A + 1}{(\sin A + \cos A + 1)(\sin A + \cos A - 1)}$

$$= \frac{2\ (\sin A + \cos A)}{(\sin A + \cos A)^2 - (1)^2}$$

$$= \frac{2(\sin A + \cos A)}{\sin^2 A + \cos^2 A + 2\sin A \cos A - 1}$$

$$= \frac{2(\sin A + \cos A)}{1 + 2\sin A \cos A - 1}$$

$$= \frac{2(\sin A + \cos A)}{2\sin A \cos A}$$

$$= \frac{\sin A + \cos A}{\sin A \cos A}$$

$$= \frac{\sin A}{\sin A \cos A} + \frac{\cos A}{\sin A \cos A}$$

$$= \frac{1}{\cos A} + \frac{1}{\sin A}$$

= sec A + cosec A

= R.H.S